An introduction to the exergy analysis of geothermal energy systems

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Abstract

In this short course, students will be introduced to the fundamentals of the exergy concept. The terms efficiency and recovery factors will be defined and discussed. Then, the students will use the data from a simplified reservoir model to estimate the pumping power requirement and heat production. Finally, they will use their estimation to calculate the theoretical and practical energy and exergy recovery factors. Finally, the CO_2 emission of the overall process will be estimated.

1 Conservation of energy

The law of conservation of energy states that the total amount of energy within an isolated domain is constant and does not change with time. In other words, energy is neither created nor destroyed. On the other hand, we constantly hear about a global *energy* problem. A simple and very relevant question that comes into mind is that if the total amount of energy is conserved over time, how can we talk about its shortage?

1.1 The quality of energy

Energy is a physical quantity that is usually defined as the ability to do work. This is not a clear definition, since work itself is defined as “energy in transition” or “energy transfer across a boundary” [3]. Energy comes in different forms, e.g., heat, mechanical energy, kinetic energy, chemical energy, etc. Chemical energy, e.g., fossil fuels, due to its high energy density or amount of energy per unit volume, is one of the most popular types of energy. This energy source is usually converted to heat via a combustion process. This heat can be used directly for warming, or can be converted to work in a heat engine. The first person who tried to quantify the limitations of the motive power of heat was Nicholas Sadi Carnot [1]. In his efforts to improve the efficiency of steam engines, he discovered a maximum limit for the amount of heat that can be converted to mechanical work, which is known today as the Carnot factor. He showed that when a certain amount of heat is transferred from a heat source to a heat sink via an ideal heat engine, only a limited fraction of the extracted heat can be converted to mechanical work. It means that different heat sources have different qualities, and that extracted heat loses its potential (to do work) when it is transferred to the heat sink (or reaches equilibrium with the heat sink). The heat sink is normally is chosen to be the environment around the heat engine. This can answer our first question: although energy is conserved, it loses its potential to do work.

The idea of quantifying the potential of energy is generalized to the other sources of energy, by defining a new thermodynamic state function. This new state function is called exergy or available energy or useful energy, which is defined as the maximum amount of work that can be extracted from a system by bringing it into equilibrium with a reference state. Various reference states can be found in the literature, which can be used for the quantification of exergy [2].

2 Conversion of energy to heat

In the previous section we touched very briefly the concept of exergy. Now, you can think about it as a measure of the quality of various energy sources. If the exergy value of an energy source is low, we can consider it as a less valuable (or less expensive) resource. We also mentioned that due to the relative convenience of handling and transferring heat, traditionally we convert other energy sources, mostly fossil fuels, and later to mechanical work and electricity.

Imagine you are given the task of warming a building, located in Delft, to a certain temperature. Normally, this temperature is around 22^oC, during a cold winter with an environmental temperature of 0^oC. As an engineer, you are always interested in finding the most economical, i.e., the cheapest, option. It means (perhaps) that you should look for the least valuable energy resource that can get the job done. You will have various alternatives, viz., a natural gas heating system, an electrical heater, an electrical heat-pump, and finally geothermal heat. Let us assume that the price of the equipments for all these methods are equal. Therefore, you need to find the method that uses the *least amount of exergy*. In the next section, we discuss a simple procedure for the quantification of the geothermal energy option.

3 Problem definition

Cold water, with a temperature of T_c [K] is being injected into a porous layer at depth L [m] through an injection well with a diameter of d_w [m]. The thickness of the porous layer is H [m] and its radius is r [m]. Hot water is produced at a temperature T_h [K]. The porosity of the reservoir is φ [m^3/m^3] and its permeability is k [m^2] (see Fig. 1).

The extracted heat from the hot water can be calculated by

$$q = \dot{Q}\rho_w c_{p,w}\left(T_h - T_c\right), \tag{1}$$

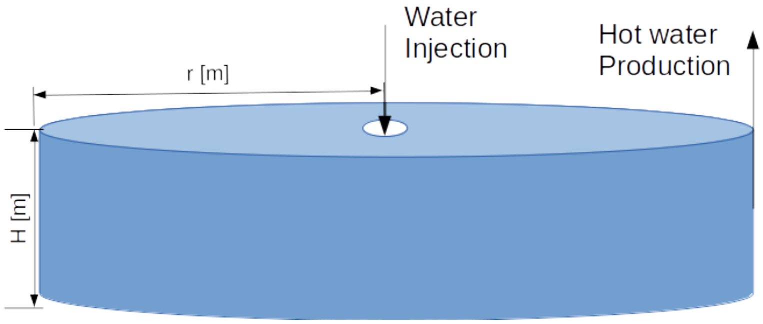

Figure 1: Schematic representation of a geothermal reservoir

where q [J/s] is the extracted heat, $\dot{Q}$ [m^3/s] is the flow rate of water, ρ_w [kg/m^3] is the density of water, and $c_{p,w}$ [J/(kg.K)] is the specific heat capacity of water.

A centrifugal pump, shown in Fig. 2, with a mechanical efficiency of η_p [-] is used to inject the cold water into the reservoir. An electrical driver with an efficiency of η_d [-] is used to drive the pump. The electricity is supplied from a natural gas power plant with an overall efficiency of η_{pp} [-]. The power (exergy) requirement of the pump is calculated by

$$Ex_p = \frac{\dot{Q}\Delta p}{\eta_p \eta_d \eta_{pp}}, \tag{2}$$

where Δp [Pa] is the summation of the pressure drops in the wells (Δp_{well} [Pa]) and the reservoir (Δp_{pm} [Pa]), i.e.,

$$\Delta p = 2\Delta p_{well} + \Delta p_{pm}. \tag{3}$$

The pressure drop in a single well can be calculated by

$$\Delta p_{well} = 4 f \rho_w \frac{L}{d_w} \frac{v^2}{2}, \tag{4}$$

where v [m/s] is the velocity of water in the well, and f [-] is the friction factor, which are calculated by

$$v = \frac{\dot{Q}}{\pi d_w^2/4}, \tag{5}$$

$$f = \begin{cases} 16/\mathrm{Re}, & \mathrm{Re} \leq 4000 \\ \frac{1}{4}\left(\frac{0.5}{\log(5.74/\mathrm{Re}^{0.9})}\right)^2, & \mathrm{Re} > 4000 \end{cases}, \tag{6}$$

$$\mathrm{Re} = \frac{\rho_w d_w v}{\mu_w}, \tag{7}$$

where μ_w [Pa.s] is the viscosity of water. Note that in Eq. (6) we assume the friction factor for the laminar regime, i.e., Re<2300, can be used on the transition regime.

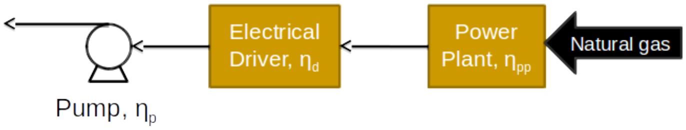

Figure 2: Fossil fuel requirement of a pump

The pressure drop in the porous medium can be estimated using Darcy's law, i.e.,

$$\Delta p_{pm} = \frac{u \mu_w r}{k}, \tag{8}$$

where u [m/s] is the Darcy velocity. Note that the above equation gives you a very rough underestimation of the pressure drop. For more realistic values of the pressure drop, use your Comsol simulation results.

3.1 CO_2 emission

Natural gas is composed of light hydrocarbons, mostly methane with the formula CH_4. Combustion of one molecule of methane emits one molecule of CO_2. The heat of combustion of methane is roughly 800 kJ/mol. Therefore, the emission factor of methane, $e_{methane}$ [kg CO_2/kJ] can be estimated as

$$e_{methane} = \frac{M_{CO_2}}{Ex_{CH_4}} = \frac{0.044}{800} = 0.055 \times 10^{-3}. \tag{9}$$

This number can be used to calculate the CO_2 emission of pumping and thus obtain a first estimation for the CO_2 emission of geothermal heat.

4 Carnot factor

The Carnot factor gives the maximum fraction of a heat source at temperature T_h [K] that can be converted to work in an environment with a temperature T_c [K]. It is calculated by

$$\eta_c = 1 - \frac{T_c}{T_h}. \tag{10}$$

Using the Carnot factor, the maximum amount of geothermal heat that can be converted to heat (or exergy)Ex_w is calculated by

$$Ex_w = \eta_c q. \tag{11}$$

5 Recovery factor and CO_2 emission

The recovery factor, R [-], for a geothermal energy extraction system is calculated by

$$R = \frac{Ex_w - Ex_p}{Ex_w}. \tag{12}$$

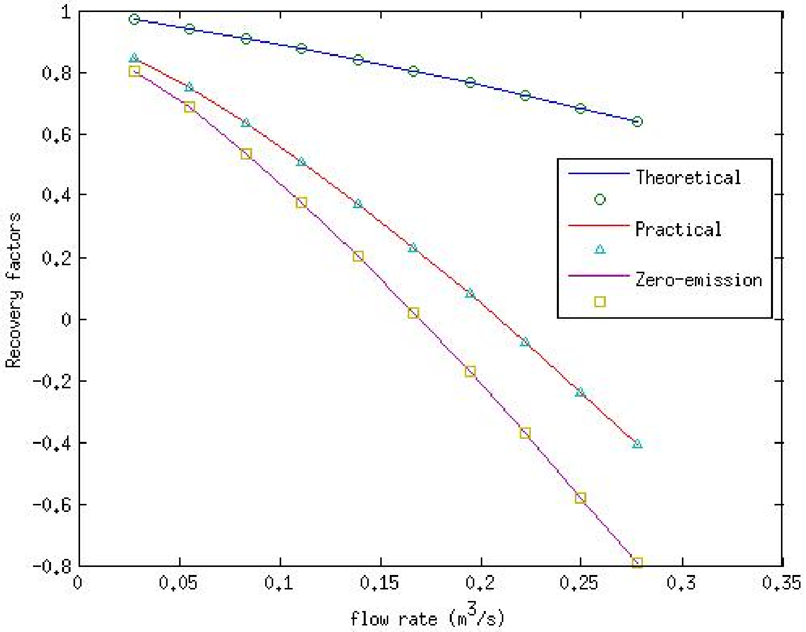

Figure 3: Recovery factor versus water flow rate for a geothermal energy system.

The CO_2 emission per unit net geothermal energy is calculated by

$$C_{geothermal} = \frac{Ex_p e_{methane}}{Ex_w}. \tag{13}$$

6 Exercise

For the geothermal energy system described above, calculate the recovery factor, R, as a function of water flow rate. Here is a sample curve that I have prepared, using a set of data given in Table 1. Moreover, you can find a Matlab code called 'geothermalexergy.m', which gives you a simple implementation of the above procedure. Your task is to modify the Matlab code, or convert it to an Excel worksheet, or follow the formulation given in this handout, and study the effect of the flow rate, well diameter, or permeability on the recovery factors and CO_2 emission. Plot the result similar to Fig. 3 and send it in with a few lines of explanations, and your matlab file or excell sheets. It is not required to include the exergy of the drilling and well completion in your calculations, but feel free to include it. Be very careful with the unit conversion.

Table 1: Data set for geothermal exergy analysis exercise

Parameter	Notation	Value	Unit	Note
Permeability	k	0.5×10^{-12}	m^2	-
Porosity	φ	0.2	m^3/m^3	-
Ambient temperature	T_0	293.15	K	-
Hot water temperature	T_h	353.15	K	-
Pump efficiency	η_p	0.8	[-]	-
Driver efficiency	η_d	0.8	[-]	-
Power plant efficiency	η_{pp}	0.4	[-]	Natural gas power plant
Well spacing	L	1000	m	-
Reservoir width	W	1000	m	-
Reservoir thickness	H	50	m	-
Water flow rate	$\dot{Q}$	100	m^3/h	-
Project life time	$t_{project}$	30	year	-
Well diameter	D_{well}	0.25	m	Inside diameter
Number of wells	n_{well}	2	[-]	-
Well tube thickness	t_{tube}	0.015	m	Steel pipe
Cement thickness	t_{cement}	0.05	m	-
Drilling exergy	ex_{drill}	70000	kJ/m	Per drilled meter
Well length	L_{well}	3000	m	Reservoir depth
Steel density	ρ_{steel}	7850	kg/m^3	-
Cement density	ρ_{cement}	2000	kg/m^3	-
Theoretical steel exergy	ex_{steel}^{th}	6738	kJ/kg	Chemical exergy of Fe
Practical steel exergy	ex_{steel}	60000	kJ/kg	-
Cement practical exergy	ex_{cement}	2500	kJ/kg	-
Methane exergy	$ex_{methane}$	800	kJ/mol	Natural gas exergy
Methane molecular weight	$M_{methane}$	0.016	kg/mol	-
Water heat capacity	$c_{p,water}$	4.1855	kJ/(kg.K)	-
Water density	ρ_{water}	1000	kg/m^3	at T_0
Water viscosity	μ_{water}	0.001	Pa.s	at T_0
		0.00035	Pa.s	at T_h
CCS exergy	ex_{ccs}	5000	kJ/kg	per kg CO_2
Methane CO_2 emission	$e_{methane}$	0.000055	kg CO_2/kJ	-

References

[1] Sadi Carnot. *Reflections on the motive power of fire: And other papers on the second law of thermodynamics.* Courier Corporation, 2012.

[2] Jan Szargut, David R Morris, and Frank R Steward. *Exergy analysis of thermal, chemical, and metallurgical processes.* Hemisphere Publishing, New York, NY, 1987.

[3] Evgeny Yantovsky, Shokotov Mykola, and J Gorski. *Zero emissions power cycles.* CRC Press, 2009.

A Matlab code

```
% an script to quantify the exergy requirement of a geothermal energy
% extraction plant clc; clear;
%% define the constants
perm = 0.5e-12; % [m^2] permeability
poros = 0.2;
T0 = 20.0+273.15; % [K] reference temperature
Th = 80.0+273.15; % [K] hot water temperature
p0 = 1.014e5; % [Pa] reference pressure
eta_pump = 0.8; % pump mechanical efficiency
eta_driver = 0.8; % electrical driver efficiency
eta_pp = 0.4; % power plant efficiency
L = 1000; % [m] well spacing
W = 1000; % [m] width of the reservoir
H = 50; % [m] thickness of the reservoir
Q = 100/3600; % [m^3/s]
t_life = 30*365*24*3600; % [s] project life time
D_well = 0.25; % [m] well diameter
nwells = 2;
t_tube = 0.015; % [m] tube thickness
t_cement = 0.050; % [m] cement thickness
ex_dril = 70000; % [kJ/m] drilling exergy
depth = 3000; % [m]
rho_steel = 7850; % [kg/m^3]
rho_cement = 2000; % [kg/m^3]
ex_steel_th = 376/0.0558; % [kJ/kg] exergy of Fe
ex_steel = 60000; %  [kJ/kg]
ex_cement = 2500; %  [kJ/kg]
ex_cao_th = 135/0.056; % [kJ/kg]
ex_methane = 800; % [kJ/mol]
cp_water = 4.1855; % [kJ/kg/K]
rho_water = 1000; % [kg/m^3]
mu_water = 1e-3; % [Pa.s]
ex_ccs = 5000; % [kJ/kg CO2]
```

```matlab
c_methane = 0.044/ex_methane; % [kg CO2/kJ]
%% quantification
i=0;
for perm = 3e-14:0.1e-14:10e-14
% for Q = 100/3600:100/3600:1000/3600
    i=i+1;
% calculation starts here ---------------------------------------------->
    m_steel = nwells*pi()/4*((D_well+2*t_tube)^2-D_well^2)*depth*rho_steel; % [kg]
    m_cement = nwells*pi()/4* ...
        ((D_well+2*t_tube+2*t_cement)^2-(D_well+2*t_tube)^2)*depth*rho_steel; % [kg]
    A_res = W*H; % [m^2] reservoir cross section
    A_pipe = pi()/4*D_well^2;
    u_res = Q/A_res; % [m/s] Darcy velocity
    u_pipe = Q/A_pipe; % [m/s] velocity of water in wells
    Re = rho_water*u_pipe*D_well/mu_water; % Reynolds number
    if Re<4000
        f = 64/Re;
    else
        f = 0.25/(log10(5.74/Re^0.9))^2;
    end
    dp_pipe = nwells*rho_water*f*depth/D_well*u_pipe^2/2; % [Pa] pressure drop in pipes
    dp_res = mu_water/perm*u_res*L; % [Pa] pressure drop in the reservoir
    Ex_th = Q*(dp_pipe+dp_res)/1000+(m_steel*ex_steel_th+ ...
        nwells*depth*ex_dril)/t_life; % [kJ] theoretical pumping exergy
    Ex_prac = Ex_th/eta_driver/eta_pp/eta_pump+(m_steel*ex_steel+ ...
        nwells*depth*ex_dril)/t_life; % [kJ] practical pumping exergy
    heat_prod = Q*rho_water*cp_water*(Th-T0); % [kJ] extracted heat
    Ex_prod = heat_prod*(1-T0/Th); % [kJ] extracted exergy
    R_th(i) = (Ex_prod-Ex_th)/Ex_prod; % theoretical recovery factor
    R_pr(i) = (Ex_prod-Ex_prac)/Ex_prod; % theoretical recovery factor
    carbon_em = Ex_prac*c_methane; % [kg CO2]
    Ex_ccs = carbon_em*ex_ccs; % [kJ] CCS exergy
    R_ze(i) = (Ex_prod-Ex_prac-Ex_ccs)/Ex_prod; % zero-emission recovery factor
    c(i) = carbon_em/Ex_prod*1000; % [kg CO2/MJ]
% end of calculation ---------------------------------------------->

end
```

```

% plot(perm, R_th, 'o', perm, R_pr, '^');
% Q = 100/3600:100/3600:1000/3600;
% plot(Q, R_th, Q, R_th, 'o', ...
%       Q, R_pr, Q, R_pr, '^', ...
%       Q, R_ze, Q, R_ze, 's');
% xlabel('flow rate (m^3/s)');
perm = 3e-14:0.1e-14:10e-14;
plot(perm, R_th, perm, R_th, 'o', ...
     perm, R_pr, perm, R_pr, '^', ...
     perm, R_ze, perm, R_ze, 's');
xlabel('Permeability [m^2]');
ylabel('Recovery factors');
legend('Theoretical', '','Practical', '', 'Zero-emission','');
```